

# Predicted Rectification and Negative Differential Spin Seebeck Effect at Magnetic Interfaces

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We study the nonequilibrium spin-thermal transport across a metal-magnetic insulator interface. The conjugate-converted thermal-spin transport is assisted by the exchange interaction at the interface, between conduction electrons in the metal lead and localized spins in the insulating magnet lead. We predict the rectification and negative differential spin Seebeck effect and resolve their microscopic mechanism. The rectification of spin Peltier effect is also identified. The phenomena predicted here are relevant for designing efficient spin/magnon diode and transistor, which could play crucial roles in controlling energy and information in functional devices.

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Recently, Spin Seebeck effect (SSE), a phenomenon that temperature bias can produce a spin current and an associated spin voltage, has been observed in magnetic metals [1], magnetic semiconductors [2, 3], magnetic insulators [4, 5] and non-magnetic materials with spin-orbit coupling [6]. Since then, the SSE has ignited a new upsurge of research, because it acts as a new method facilitating the functional use of heat and opens a new possibility of spintronics [7] and spin caloritronics [8, 9].

Of particular interest is the SSE in the insulating magnetic interface [4, 5]. The reason is that different from spin-dependent Seebeck effect, SSE allows heat to generate a pure flow of spin angular momentum, a flow of spins *without* electron currents. This becomes obvious only after the observation of the SSE through magnetic insulator-metal interfaces [4, 5]. Itinerant electrons are often problematic in the thermal design of devices, of which the issue can be avoided by the SSE in magnetic insulator interfaces without conducting electron currents. It allows us to construct efficient thermoelectric devices upon new principles [10] and to realize non-dissipative information and energy transfer in the absence of Joule heating [11, 12].

In this Letter, we predict new phenomena of the SSE across metal-insulating magnet interface: the *rectification* and *negative differential* SSE. That is, reversing the thermal bias gives asymmetric spin currents and increasing thermal bias gives a decreasing spin current. The rectification of spin Peltier effect (SPE) is also uncovered. We first demonstrate these interesting effects in a nanoscale magnetic interface system. Our simple model readily renders analytic interpretations and clear physical insights of their microscopic mechanism. We then turn to a macroscopic magnetic interface to demonstrate that the nontrivial rectification and negative differential SSEs are absent in such macroscopic system. Conditions to retain these intriguing properties are also discussed. The phenomena predicted here are relevant for designing efficient spin/magnon diode and transistor, which could play crucial roles in spintronics [7], spin caloritronics [9] and magnonics [13], and could have potential applications in controlling energy and information in low-dimensional nanodevices [14].

The system we are studying is schematically illustrated in Fig. 1, similar to the setup of longitudinal spin Seebeck exper-

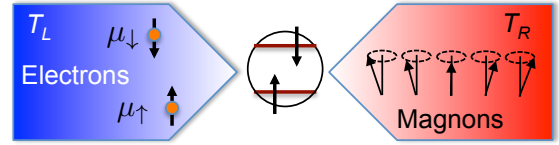


FIG. 1. Schematic illustration of the two-level system setup that describes the nanoscale magnetic interface.

iment [5]. We consider a nanoscale interface as in the situation of quantum point contacts. As such, the electron states near the interface can be simplified as impurity states or local states on a quantum dot:  $H_C = \varepsilon_\uparrow d_\uparrow^\dagger d_\uparrow + \varepsilon_\downarrow d_\downarrow^\dagger d_\downarrow$ , where  $d_\sigma^\dagger (d_\sigma)$  is the creation (annihilation) operator of the local electron with spin  $\sigma$  and energy  $\varepsilon_\sigma$  at the interface.

The left metallic lead is described in terms of the free electron gas:  $H_L = \sum_{k\sigma} (\varepsilon_{k\sigma} - \mu_\sigma) c_{k\sigma}^\dagger c_{k\sigma}$ , ( $\sigma = \uparrow, \downarrow$ ), with possibly different spin-dependent chemical potentials  $\mu_\sigma$  induced by spin accumulation [15, 16]. The spin voltage  $\mu_\downarrow - \mu_\uparrow$  can be measured by Hanle method [3] or be converted into an electric voltage through the inverse spin Hall effect [4, 5]. The electrons are freely exchanged between the left metal and the central quantum dot without spin flip:  $V_L = \sum_{k\sigma} t_{k\sigma} c_{k\sigma}^\dagger d_\sigma + H.c.$

The right insulating magnetic lead is described by a Heisenberg lattice:

$$H_R = -J \sum_{\langle i,j \rangle} [\frac{1}{2} S_i^+ S_j^- + \frac{1}{2} S_i^- S_j^+ + S_i^z S_j^z], \quad (1)$$

where  $S_j^\pm$  is the raising (lowering) operator for the localized spin at site  $j$ ,  $S_j^z$  is the spin operator of the  $z$  direction and  $J$  denotes the exchange coupling strength. It is convenient to map the spin operators into bosonic magnons by Holstein-Primakoff transformation [12, 17]:  $S_j^+ = \sqrt{2S_0 - a_j^\dagger a_j} a_j$ ,  $S_j^- = a_j^\dagger \sqrt{2S_0 - a_j^\dagger a_j}$ ,  $S_j^z = S_0 - a_j^\dagger a_j$ , where  $S_0$  is the length of localized spins. Clearly, the creation (annihilation) of a local magnon  $a_j^\dagger (a_j)$  at site  $j$  corresponds to the lowering (rising) of the local spin component, i.e., excitation of magnons means that spins point less in  $z$ -direction and magnetization goes down. At large spin limit or low temperatures ( $\langle a_j^\dagger a_j \rangle \ll 2S_0$ ) we can approximate  $S_j^- \approx \sqrt{2S_0} a_j^\dagger$  and

$S_j^+ \approx \sqrt{2S_0} a_j$ . Therefore, after a Fourier transform into the momentum space, the right insulating magnetic lead is approximated by the free magnon gas:

$$H_R \approx \sum_q \hbar \omega_q a_q^\dagger a_q + \text{const.}, \quad (2)$$

where the dispersion of  $\omega_q$  depends on the lattice details that are irrelevant to our present problem.

Similar to Refs. [18–20], we describe the electron-spin interaction at the nanoscale magnetic interface by the  $s$ - $d$  exchange coupling [21]:

$$V_R = - \sum_q J_q [S_q^z (d_{\uparrow}^\dagger d_{\uparrow} - d_{\downarrow}^\dagger d_{\downarrow}) + S_q^- d_{\uparrow}^\dagger d_{\downarrow} + S_q^+ d_{\downarrow}^\dagger d_{\uparrow}], \quad (3)$$

where  $J_q$  denotes the effective exchange coupling at the interface,  $S_q^- \approx \sqrt{2S_0} a_q^\dagger$  and  $S_q^+ \approx \sqrt{2S_0} a_q$ . It is clear that the first term just splits the two local energy levels at interface so that its contribution can be absorbed into a renormalized  $\varepsilon_\sigma$  (we keep the same notation) and we have the spin splitting  $\varepsilon_\downarrow > \varepsilon_\uparrow$  generally. The second term  $S_q^- d_{\uparrow}^\dagger d_{\downarrow}$  describes the magnon emission process (into the right lead) associated with scattering of a spin-down electron to a spin-up electron at the interface. The last term  $S_q^+ d_{\downarrow}^\dagger d_{\uparrow}$  describes the reversed process that the spin-up electron in the dot absorbs a magnon from the right lead and excites to the upper level with spin-flip.

Considering each magnon carries an angular momentum of  $-\hbar$  (associated with a magnetic moment), the magnon current is equivalent to a spin-down current, which is then obtained by the Heisenberg equation of motion  $I_S := \frac{i}{\hbar} \langle [\frac{d_{\downarrow}^\dagger d_{\downarrow} - d_{\uparrow}^\dagger d_{\uparrow}}{2}, V_R] \rangle$  or  $I_S := \frac{i}{\hbar} \langle [V_R, \sum_q a_q^\dagger a_q] \rangle$ . From either definition, we can get the same spin (magnon) current:

$$I_S = \frac{i}{\hbar} \sum_q J_q \left( \langle S_q^- d_{\uparrow}^\dagger d_{\downarrow} \rangle - \langle S_q^+ d_{\downarrow}^\dagger d_{\uparrow} \rangle \right). \quad (4)$$

Notice that the expectation value  $\langle S_q^- d_{\uparrow}^\dagger d_{\downarrow} \rangle$  is just the complex conjugate of  $\langle S_q^+ d_{\downarrow}^\dagger d_{\uparrow} \rangle$ , as a next step, one has only to evaluate the equation of motion:

$$\begin{aligned} \frac{d}{dt} \langle S_q^+ d_{\downarrow}^\dagger d_{\uparrow} \rangle &= \frac{i}{\hbar} \langle [H_C + V_R + H_R, S_q^+ d_{\downarrow}^\dagger d_{\uparrow}] \rangle \\ &= \frac{i}{\hbar} (\varepsilon_\downarrow - \varepsilon_\uparrow - \hbar \omega_q) \langle S_q^+ d_{\downarrow}^\dagger d_{\uparrow} \rangle + \frac{i}{\hbar} J_q \langle [S_q^+ d_{\downarrow}^\dagger d_{\uparrow}, S_q^- d_{\uparrow}^\dagger d_{\downarrow}] \rangle. \end{aligned} \quad (5)$$

Note that the argument  $t$  of each operator is hidden for clarity. Following Ref. [22], in the steady state of large  $t$  limit, one can write  $\langle S_q^+ d_{\downarrow}^\dagger d_{\uparrow} S_q^- d_{\uparrow}^\dagger d_{\downarrow} \rangle = \langle S_q^+ S_q^- \rangle \langle d_{\downarrow}^\dagger d_{\downarrow} \rangle \langle d_{\uparrow}^\dagger d_{\uparrow} \rangle = 2S_0(1 + N_R) f_{L\downarrow}(1 - f_{L\uparrow})$  where  $N_R = [e^{\hbar \omega_q / (k_B T_R)} - 1]^{-1}$  is the magnon distribution function in the right lead, which acts as a thermal bath with temperature  $T_R$ ;  $f_{L\sigma} = [e^{(\varepsilon_\sigma - \mu_\sigma) / (k_B T_L)} + 1]^{-1}$  denotes the electron distribution function at the central dot, which is in equilibrium with the left lead at temperature  $T_L$ . Similarly,  $\langle S_q^- d_{\uparrow}^\dagger d_{\downarrow} S_q^+ d_{\downarrow}^\dagger d_{\uparrow} \rangle = \langle S_q^- S_q^+ \rangle \langle d_{\downarrow}^\dagger d_{\downarrow} \rangle \langle d_{\uparrow}^\dagger d_{\uparrow} \rangle =$

$2S_0 N_R f_{L\uparrow}(1 - f_{L\downarrow})$ . Thus, by applying the Markov condition in the steady state  $t \rightarrow \infty$ , one obtains:  $\langle S_q^+ d_{\downarrow}^\dagger d_{\uparrow} \rangle = \frac{i}{\hbar} 2S_0 J_q \int_0^\infty d\tau e^{i(\varepsilon_\downarrow - \varepsilon_\uparrow - \hbar \omega_q)\tau / \hbar} [f_{L\downarrow}(1 - f_{L\uparrow})(1 + N_R) - f_{L\uparrow}(1 - f_{L\downarrow})N_R]$ . Substituting it into Eq. (4), we finally arrive at the expression of spin current:

$$I_S = \frac{2S_0}{\hbar} \Gamma_J [f_{L\downarrow}(1 - f_{L\uparrow})(1 + N_R) - f_{L\uparrow}(1 - f_{L\downarrow})N_R], \quad (6)$$

where  $\Gamma_J = 2\pi \sum_q J_q^2 \delta(\varepsilon_\downarrow - \varepsilon_\uparrow - \hbar \omega_q)$ .

This expression is familiar from the results of applying Fermi's golden rule to the definition Eq. (4). The first product  $f_{L\downarrow}(1 - f_{L\uparrow})(1 + N_R)$  describes the relaxation rate of the occupied higher spin-down state flipping to the empty lower spin-up state with emitting a magnon with energy  $\hbar \omega_q$  into the right lead. The second product  $f_{L\uparrow}(1 - f_{L\downarrow})N_R$  reversely describes the excitation rate of the occupied lower spin-up state flipping to the empty higher spin-down state with absorbing a magnon from the right lead. The two spin-flip processes, accompanied by the magnon emission/absorption, conserve not only the angular momentum, but also the energy, which is imposed by the delta function  $\delta(\varepsilon_\downarrow - \varepsilon_\uparrow - \hbar \omega_q)$ . In this way, only magnons with energy  $\hbar \omega_q = \varepsilon_\downarrow - \varepsilon_\uparrow$  is able to transfer through the magnetic interface. Therefore, for the energy current carried by magnons, we can also obtain similarly:

$$I_Q = \frac{2S_0}{\hbar} \Gamma_J (\varepsilon_\downarrow - \varepsilon_\uparrow) [f_{L\downarrow}(1 - f_{L\uparrow})(1 + N_R) - f_{L\uparrow}(1 - f_{L\downarrow})N_R]. \quad (7)$$

Considering  $\mu_{\downarrow, \uparrow} = \mu_0 \pm \Delta\mu_s/2$ ,  $T_{L,R} = T_0 \pm \Delta T/2$ , the thermal-spin transport coefficients are conventionally considered in the linear response regime: expanding to the first order of spin voltage bias  $\Delta\mu_s = \mu_\downarrow - \mu_\uparrow$  and thermal bias  $\Delta T = T_L - T_R$ , which yields

$$\begin{pmatrix} I_S \\ I_Q \end{pmatrix} = \begin{pmatrix} \mathcal{G} & \mathcal{G} S T_0 \\ \mathcal{G} \Pi & \kappa T_0 \end{pmatrix} \begin{pmatrix} \Delta\mu_s \\ \Delta T / T_0 \end{pmatrix}, \quad (8)$$

where  $\mathcal{G} = \frac{S_0 \Gamma_J}{\hbar T_0} / (\sinh[\frac{\varepsilon_\downarrow - \mu_0}{k_B T_0}] - \sinh[\frac{\varepsilon_\uparrow - \mu_0}{k_B T_0}] + \sinh[\frac{\varepsilon_\downarrow - \varepsilon_\uparrow}{k_B T_0}])$  is the spin conductance, generated by the spin voltage difference.  $\kappa = \mathcal{G}(\varepsilon_\downarrow - \varepsilon_\uparrow)^2 / T_0$  denotes the heat conductance, produced by the temperature bias.  $\mathcal{S} = -\Delta\mu_s / \Delta T|_{I_S=0} = (\varepsilon_\downarrow - \varepsilon_\uparrow) / T_0$  is the spin Seebeck coefficient, depicting the power of generating spin voltage by the temperature bias.  $\Pi = I_Q / I_S|_{\Delta T=0} = \varepsilon_\downarrow - \varepsilon_\uparrow$  is the spin Peltier coefficient, depicting the power of heating or cooling carried by per unit spin current. It is seen clearly that we have  $\Pi = S T_0$  so that the Onsager reciprocal relation  $\mathcal{G} S T_0 = \mathcal{G} \Pi$  is fulfilled in the present system [23].

In the linear response regime, the spin Seebeck and Peltier coefficients are symmetric when reversing  $\Delta\mu_s \rightarrow -\Delta\mu_s$  and  $\Delta T \rightarrow -\Delta T$ . However, as we will see soon, when we go to the nonlinear response regime, the rectification of SPE and SSE will emerge. In some cases, we can even have the negative differential SSE.

We first examine the SPE which gives the magnonic heat current by the spin voltage. Figure 2 illustrates the rectification of SPE in our nanoscale interface system: the positive spin voltage produces a large heat current, thus acting

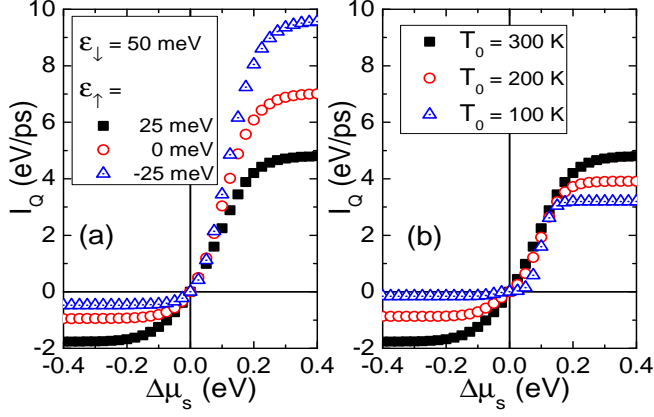


FIG. 2. Rectification of spin Peltier effect. (a) Heat current  $I_Q$  as a function of spin voltage  $\Delta\mu_s$  with  $\mu_{\downarrow,\uparrow} = \mu_0 \pm \Delta\mu_s/2$ , for varying  $\epsilon_{\downarrow,\uparrow}$ .  $\mu_0 = 0$ ,  $T_L = T_R = T_0 = 300$  K. (b) Heat current as a function of spin voltage for varying  $T_0$ .  $\epsilon_\downarrow = 50$  meV,  $\epsilon_\uparrow = 25$  meV,  $\mu_0 = 0$ . For all calculations in this work, we set  $\Gamma_J = 2.5$  meV,  $S_0 = 16$  [19].

as a good spin-heater or spin-cooler; but reversing the spin voltage gives a less heat current, thus acting as a bad spin-heater/cooler or even a spin-thermal insulator. Since  $I_S$  is proportional to  $I_Q$ ,  $I_S$  vs  $\Delta\mu_s$  will exhibit the similar behavior, thus acting as a spin current diode.

The rectification of SPE can be understood in the following: when the spin voltage is positive ( $\mu_\downarrow > \mu_\uparrow$ ), the electron prefers occupying the higher spin-down state and emptying the lower spin-up state. Thus, the transfer is dominated by the magnon emission accompanied by the relaxation of spin-down flipping to the spin-up state, so that we have  $I_Q \approx \frac{2S_0}{\hbar} \Gamma_J (\epsilon_\downarrow - \epsilon_\uparrow) (1 + N_R)$  from Eq. (7). When the spin voltage is reversed, the electron prefers occupying the lower spin-up state and emptying the higher spin-down state. Then, the transfer is dominated by the magnon absorption accompanied by the excitation of spin-up flipping to the spin-down state, so that we have  $I_Q \approx -\frac{2S_0}{\hbar} \Gamma_J (\epsilon_\downarrow - \epsilon_\uparrow) N_R$  with the negative sign denoting the reversed current direction. Therefore, the rectification ratio is obtained as  $(1 + N_R)/N_R = \exp(\frac{\epsilon_\downarrow - \epsilon_\uparrow}{k_B T_0})$ . This indicates that increasing the spin-splitting  $\epsilon_\downarrow - \epsilon_\uparrow$  or decreasing the temperature  $T_0$  can enhance the rectification of SPE, which is consistent with Fig. 2 (a) and (b), respectively. Note that heat and spin currents prefer  $\Delta\mu_s > 0$  ( $\mu_\downarrow > \mu_\uparrow$ ) is a consequence of the pre-defined up-magnetization direction of the insulating magnetic lead. Reversing the pre-defined magnetization direction (as well the central two levels) will reverse the rectification.

We next examine the SSE which generates the spin current by the thermal bias. Figure 3 shows the rectification of SSE clearly that the thermal-induced spin currents are asymmetric with respect to reversing the temperature bias. In particular, when the two levels are either both above the chemical potential  $\epsilon_\downarrow > \epsilon_\uparrow > \mu_0$  [see case (i) in Fig. 3(a)] or both below  $\mu_0 > \epsilon_\downarrow > \epsilon_\uparrow$  [see case (iv) in Fig. 3(a)], we can even have *negative differential SSE*: when increasing the temper-

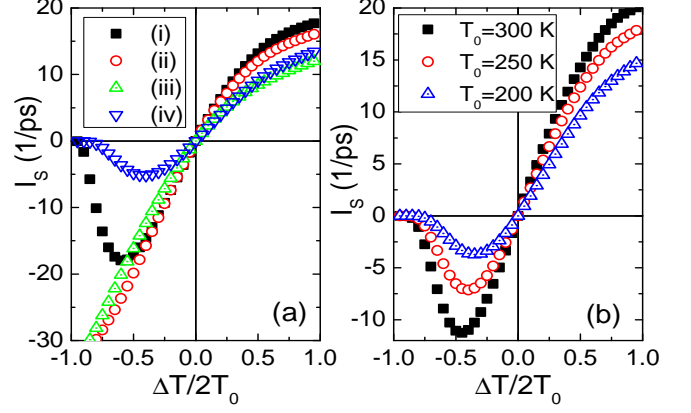


FIG. 3. Rectification and negative differential spin Seebeck effect. (a) Magnon spin current  $I_S$  as a function of the normalized temperature bias  $\Delta T/2T_0$  with  $T_{L,R} = T_0 \pm \Delta T/2$ , for varying  $\epsilon_{\downarrow,\uparrow}$ : (i)  $\epsilon_\downarrow = 50$  meV,  $\epsilon_\uparrow = 10$  meV; (ii)  $\epsilon_\downarrow = 50$  meV,  $\epsilon_\uparrow = 0$ ; (iii)  $\epsilon_\downarrow = 50$  meV,  $\epsilon_\uparrow = -25$  meV; (iv)  $\epsilon_\downarrow = -25$  meV,  $\epsilon_\uparrow = -75$  meV. Other parameters are  $\mu_\downarrow = \mu_\uparrow = 0$ ,  $T_0 = 300$  K. (b) Magnon spin current as a function of the normalized temperature bias, for varying  $T_0$ .  $\epsilon_\downarrow = 50$  meV,  $\epsilon_\uparrow = 25$  meV,  $\mu_\downarrow = \mu_\uparrow = 0$ .

ature difference  $T_R - T_L = -\Delta T$ , instead of observing an increasing spin current as in linear response regime, we get a decreasing and even vanishing spin current from the right magnetic insulator to the left metallic lead. Figure 3(b) shows that the negative differential SSE also exists for a wild range of temperatures.

The emergence of negative differential SSE can be reasoned as follows: when  $T_R > T_L$ , the thermal bias drives spin current from the right to the left. If near the linear response regime, it is natural to have a positive differential SSE that lowering  $T_L$  will increase the thermal bias which in turn increases the spin current. If we further decrease  $T_L$  with assuming two levels are both above (below) the chemical potential, the two states will be both depleted (occupied), which in turn severely suppresses the magnon emission/absorption process that requires the concurrence of one occupied and one empty state. As a consequence, the conductance will decrease although the bias increases. When the conductance decreases faster than the increasing bias, the negative differential SSE emerges.

Finally, we turn to examine the possibility of rectification and negative differential SPE and SSE in macroscopic magnetic interfaces. In this situation, the two-level central system mimicking the nanoscale interfaces is discarded. As such, the left metal  $H_L$  interacts with the right magnetic insulator  $H_R$  directly, with the interfacial electron-spin interaction:

$$H_{sd} = - \sum_{k,q} J_q [S_q^- c_{k\uparrow}^\dagger c_{k+q\downarrow} + S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow}], \quad (9)$$

where the term  $-J_q S_q^z (c_{k\uparrow}^\dagger c_{k\uparrow} - c_{k\downarrow}^\dagger c_{k\downarrow})$  is customarily absorbed into  $H_L$ . Following similar procedures as above, from the definitions  $I_S = \frac{i}{\hbar} [H_{sd}, \sum_q a_q^\dagger a_q]$  and  $I_Q =$

$\frac{i}{\hbar}[H_{sd}, \sum_q \hbar\omega_q a_q^\dagger a_q]$ , we obtain:

$$I_S = \frac{2S_0}{\hbar} \int_0^\infty d\omega F_R(\omega) \int_{-\infty}^\infty d\varepsilon \rho_L(\varepsilon) \mathcal{W}(\varepsilon, \omega), \quad (10)$$

$$I_Q = \frac{2S_0}{\hbar} \int_0^\infty d\omega F_R(\omega) \hbar\omega \int_{-\infty}^\infty d\varepsilon \rho_L(\varepsilon) \mathcal{W}(\varepsilon, \omega), \quad (11)$$

where  $\mathcal{W}(\varepsilon, \omega) = f_{L\downarrow}(\varepsilon + \hbar\omega)[1 - f_{L\uparrow}(\varepsilon)][1 + N_R(\hbar\omega)] - f_{L\uparrow}(\varepsilon)[1 - f_{L\downarrow}(\varepsilon + \hbar\omega)]N_R(\hbar\omega)$  with  $f_{L\sigma}(x) = [e^{(x-\mu_\sigma)/(k_B T_L)} + 1]^{-1}$  and  $N_R(\hbar\omega) = [e^{\hbar\omega/(k_B T_R)} - 1]^{-1}$ . Note  $\mathcal{W}$  has the same physical meaning as discussed after Eq. (6). The only difference is that we now have one integral of magnon energy and one integral of electron energy for the macroscopic interface.  $F_R(\omega)$  contains the bulk magnon density of state (DOS) and the electron-spin couplings, which is reminiscent of Eliashberg function in the field of electron-phonon coupling [21].  $\rho_L(\varepsilon)$  denotes the bulk electron DOS in the left metal lead.

It is customary and legitimate to take a constant bulk electron DOS  $\rho_L(\varepsilon) \approx \rho_L(\mu_0)$  because for a good metal, the integral over  $d\varepsilon$  converges within a thermal energy  $k_B T_L$  around the chemical potential [21]. Then, by applying the equality  $\int d\varepsilon f_{L\downarrow}(\varepsilon + \hbar\omega)[1 - f_{L\uparrow}(\varepsilon)] = (\hbar\omega - \Delta\mu_s)/[e^{(\hbar\omega - \Delta\mu_s)/(k_B T_L)} - 1]^{-1} = (\hbar\omega - \Delta\mu_s)N_L(\hbar\omega - \Delta\mu_s)$ , we finally arrive at the formulas

$$I_S = \frac{2S_0\rho_L(\mu_0)}{\hbar} \int_0^\infty d\omega F_R(\omega)(\hbar\omega - \Delta\mu_s) \times [N_L(\hbar\omega - \Delta\mu_s) - N_R(\hbar\omega)]; \quad (12)$$

$$I_Q = \frac{2S_0\rho_L(\mu_0)}{\hbar} \int_0^\infty d\omega F_R(\omega)\hbar\omega(\hbar\omega - \Delta\mu_s) \times [N_L(\hbar\omega - \Delta\mu_s) - N_R(\hbar\omega)]. \quad (13)$$

Clearly, in the absence of thermal bias  $T_L = T_R$ , the rectification of SPE exists, i.e., the heat current  $I_Q$  is asymmetric under reversing the spin voltage  $\Delta\mu_s \rightarrow -\Delta\mu_s$ . Similarly,  $I_S$  is also asymmetric under reversing the spin voltage. Moreover, it is straightforward to verify that  $\partial_{\Delta\mu_s} I_S$  and  $\partial_{\Delta\mu_s} I_Q$  are both positive. Therefore the negative differential SPE is absent. These observations are the same as above in the nanoscale magnetic interface.

For SSE, we take  $\Delta\mu_s = 0$  and  $T_L \neq T_R$ . In this way, Eqs. (12, 13) reduce to Landauer-type formulas

$$I_S = \frac{2S_0\rho_L(\mu_0)}{\hbar} \int_0^\infty d\omega F_R(\omega)\hbar\omega [N_L(\omega) - N_R(\omega)]; \quad (14)$$

$$I_Q = \frac{2S_0\rho_L(\mu_0)}{\hbar} \int_0^\infty d\omega F_R(\omega)(\hbar\omega)^2 [N_L(\omega) - N_R(\omega)]. \quad (15)$$

Since the temperature dependence only manifests in Bose distributions  $N_L$  and  $N_R$ , one can readily verify that for this case of bulk macroscopic magnetic interfaces we can never have the rectification and negative differential SSE.

One may wonder how is the conclusion of the macroscopic magnetic interface so different from that of the nanoscale interface. From the above analysis, one can find that the constant bulk electron DOS is the crucial assumption that leads

to the Landauer-type Eqs. (14, 15) from Eqs. (10, 11). In low-dimensional nanoscale systems, the DOS should not be a smooth function but vary strongly on energy. Therefore, when the magnetic interface scales down to the nanoscale, we have to keep the electron DOS  $\rho_L(\varepsilon)$  inside the energy integral. In this way, we will not have the Landauer-type equations but instead retain the intriguing properties of rectification and negative differential SSE.

For example, considering a common Lorentzian-type DOS  $\rho_L(\varepsilon) = \frac{1}{\pi} \frac{\Gamma}{(\varepsilon - \varepsilon_0)^2 + \Gamma^2}$  where the half-width  $\Gamma$  is small, the resonant peak of DOS becomes sharp at  $\varepsilon_0$ , so that Eqs. (10, 11) reduce to

$$I_S = \frac{2S_0}{\hbar} \int_0^\infty d\omega F_R(\omega) \mathcal{W}(\varepsilon_0, \omega), \quad (16)$$

$$I_Q = \frac{2S_0}{\hbar} \int_0^\infty d\omega F_R(\omega) \hbar\omega \mathcal{W}(\varepsilon_0, \omega). \quad (17)$$

These results are familiar from Eqs. (6, 7). The mere difference is that the currents now have an integral over the magnon spectrum, instead of the resonant transfer of a single-mode magnon. It is straightforward to verify that Eqs. (16, 17) have similar behaviors of the rectification and negative differential SSE as Eqs. (6, 7).

Generally, the Eliashberg-like function  $F_R(\omega)$  also has electronic energy dependence [21]. By considering non-smooth, strong energy dependent  $F_R(\varepsilon, \omega)$  in some designed magnetic interfacial systems, we expect to have the similar rectification and negative differential transport effects.

In summary, we have studied the nonequilibrium spin-thermal transport across a metal-magnetic insulator interface. The conjugate-converted thermal-spin transport is assisted by the exchange interaction at the interface, between conduction electrons in the metal lead and localized spins in the insulating magnet lead. We have predicted the rectification and negative differential SSE as well as the rectification of SPE, and resolved their microscopic mechanism. Since the heat current is proportional to the magnon spin current, we also possess the rectification of spin current driven by spin voltage, and the rectification and negative differential thermal conductance driven by the temperature bias, as in dielectric phononics [14].

It is also readily to generalize our findings to the interfaces of metal-magnetic metal/semiconductor. Since recent studies imply the important role of phonon-drag in SSE [6, 24–27], taking account of the effect of nonequilibrium phonons on rectification and negative differential SSE would be an interesting topic. By integrating the phononics [14] with spintronics [7], spin caloritronics [9] and magnonics [13], we expect to possess more opportunities to achieve smart control of energy and information in low-dimensional nanodevices.

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